

The Three Faces of a Fixed Point

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Abstract

It has been argued that the only mathematically precise quantum descriptions of gravitating systems are from vantage points which allow an unbounded amount of information to be gathered. For an eternally inflating universe that means a hat, i.e., the asymptotic future of a flat FRW universe. The boundary of the hat (the place where it enters the bulk geometry) is the seat of the FRW/CFT duality. In this paper we discuss the perturbative and non-perturbative fixed points of FRW/CFT as seen from the three regions which share this boundary.

Perturbatively, there is nothing universal about the FRW duality; there is a separate construction for each possible initial vacuum. We explain how bubble collisions induce a non-perturbative flow to a universal fixed point which contains information about the entire landscape. We also argue that there is a duality between the landscape of de Sitter vacua and a discretuum of very low-dimension relevant operators in the FRW/CFT spectrum. In principle this provides us with a precise definition of unstable de Sitter vacua.

Along the way we show that BPS domain walls play a special role in reinforcing the “persistence of memory” and breaking the symmetries of the hat.

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1 Introduction

Conventional bulk gravitational degrees of freedom are useful for many purposes, but they are inadequate for others. They clearly fail for certain questions involving quantum information in the presence of event horizons. The origin of black hole entropy is only one example. Alternative formulations, in so far as they are known, are holographic [1].

It is not obvious whether any of the problems of cosmology are so subtle as to require the precision of a holographic description. True, inflation and accelerated expansion imply the existence of event horizons, and primordial density fluctuations appear to be born

out of quantum fluctuations. But successful descriptions of these phenomena seem to be straightforward enough that the subtleties of the holographic principle, and of horizon microstates, appear to be irrelevant. So one might reasonably ask: Who needs a microscopic holographic description?

The answer may be that the multiverse does. The problem of putting a probability measure [2] on the space of vacua has so far defied current formulations of eternal inflation. It's not that promising phenomenological candidates don't exist—they do—but the principles underlying them do not.

Many questions suggest themselves. Among them:

- Is the global multiverse¹ described by a single wave function?
- Does probability theory need revision in view of the paradoxes of exponentially increasing populations [3].
- Does quantum mechanics make sense on scales so large that no observer can ever collect data and confirm statistical predictions? A simple example would be the correlation function between vacua at causally disconnected points [4].
- Are causally disconnected degrees of freedom independent, entangled, or are they complementary descriptions of the same thing?
- What is the precise meaning of a metastable vacuum? In scattering theory a metastable state is a pole in the complex energy plane. What is the mathematical framework in which metastable de Sitter space is defined?
- Is the measure problem so delicate and subtle that resolving it requires understanding the above items?

It seems clear that some new principle is needed to understand these questions. One conjecture, based on its validity in AdS and Minkowski examples, is that the only precise representation of the multiverse involves a holographic platform from which an observer's past light-cone encompasses an infinite entropy bound [5, 6, 7].² Infinite information capacity would be necessary to encode the entire multiverse or even a small comoving portion of

¹In this context the term multiverse could stand for any geometry or cosmology that contains causally disconnected regions.

²This conjecture also fits well with ideas about the interpretation of quantum mechanics expressed in [4].

it. The only candidates in an eternally inflating multiverse seem to be observers that enter stable supersymmetric bubbles which approach $\Lambda = 0$ at late times.³ Such observers are called “census takers” [8, 6, 9], and they seem to possess the capability to process arbitrarily large amounts of information.⁴ According to this line of thinking, the key to information-theoretic puzzles (possibly including the measure problem) is first understanding how to organize the observations of a census taker into a well-defined mathematical theory, and second in clarifying how the information about the larger multiverse is encoded in these observations.

The perturbative observations of a census taker have been studied in a series of papers [8, 10, 9, 11], which have established a local conformal structure on the census-taker’s cosmic sphere. This program is far from complete, but the results so far are consistent with a conjectural “FRW/CFT” duality that realizes a census taker’s observations as correlation functions in a CFT coupled to gravity living on his/her cosmic sphere.⁵ These perturbative calculations have a rather unappealing feature in that they seem to depend on the particular vacuum the census taker ends up in and also on the vacuum he/she nucleated from. Moreover they have so far not been able to shed light on how to extract information about the larger multiverse. In this paper we will claim that once non-perturbative bubble nucleation and thermal fluctuation are taken into account, there is a universal fixed point describing the late-time observations of all census takers and encoding information about the entire landscape.⁶ This argument has been made before [6, 9], but we are now able to be much more specific about it.

³We expect supersymmetry to be needed for the precise cancellation that sets Λ to zero. The only exception to this, pointed out by Eva Silverstein, seems to be bubbles where the coupling asymptotically goes to zero.

⁴A subtlety here is that any “actual” census taker has physical limitations that allow only a finite amount of information to be received and stored, but the point is that there is no limit on how large this amount can be. Moreover nothing in this paper really depends on the existence of actual census takers, below we define them simply as time-like curves. We thank Edward Witten for many discussions of these issues.

⁵The name “FRW/CFT” turns out to be somewhat inaccurate, in that non-perturbative effects in the multiverse will break the conformal symmetry. This is somewhat analogous to the term “AdS/CFT”, which has since been widely applied to systems that are not conformal. In our case a more accurate name might be “eternal inflation/fractal flow” duality, using a term we will introduce below, but this is rather tedious so we will continue to use “FRW/CFT” with the understanding that it stands for a more general set of ideas.

⁶More precisely we will say that each connected moduli space of $\Lambda = 0$ vacua has a single type of census taker with a unique fixed point. If there are disconnected moduli spaces then it seems likely that each has its own fixed point. The case of moduli spaces that are connected only at infinity is subtle, we comment briefly on it below.

We will also focus on the existence and importance BPS domain walls, connecting the FRW region and a bubble of negative cosmological constant [12]. If such domain walls exist they significantly modify both the boundary theory and the evolution within the FRW bulk geometry. We will see that such domain walls “shred” the census taker’s sky and turn it into a fractal of measure zero, breaking the conformal symmetry to an unusual kind of scale invariance. This breaking is a form of “persistence of memory”, which although it sounds threatening, has the positive effect of selecting a preferred frame of reference and eliminating the ambiguity in the choice of time-variable.

In making our arguments, we will make constant use of a partition of the multiverse into three separate regions. The three regions are defined by their causal relationship to the census taker, and the interplay between them is a new crucial part of the FRW/CFT duality. We refer to the complementary descriptions of the three regions within the dual theory as the three faces of the fixed point.

2 Three Regions

We begin by giving general definitions of some of the standard FRW/CFT concepts.⁷

- A “census taker” is a timelike curve of infinite future extent, whose nearby geometry approaches a piece of Minkowski space at late times.

Given a particular census taker, we can divide up spacetime into three regions based on their causal relation to that census taker.

- The causal past of the census taker, meaning all points which can influence the census taker’s worldline by sending in massless particles on null geodesics, we denote as \mathcal{C} .
- A spacetime point is in “region III” if it is not in \mathcal{C} .
- A spacetime point in \mathcal{C} which can influence some point in region III is in “region II”.
- A spacetime point in \mathcal{C} which is not in region II is in “region I”.

In [8] [9] the emphasis was on the region I, but in this paper we will be interested in all three regions. We can similarly partition the future boundary as follows.

- The census taker’s “hat” is the future boundary of region I.

⁷These geometric definitions are really just semiclassical notions, but the idea is that they will enable a bulk discussion that suggests a precise boundary version.

- The “rim” of the hat, which we denote Σ , is the future boundary of region II.
- The “fractal future” is the future boundary of region III.

Clearly Σ is a special part of the boundary, it is the interface between the census taker’s observations and the fractal future. Σ is the seat of the FRW/CFT duality.

These definitions can be applied to any spacetime that has a census taker; for example in empty Minkowski space only region I exists, while for an eternal black hole in Minkowski space region III is behind the horizon and the rest of the space is region II. For an evaporating black hole all three regions exist.

2.1 The Coleman De Luccia Geometry

The intuitive meaning of the three regions in the multiverse is best illustrated by considering the example of the Coleman De Luccia spacetime describing a bubble of Minkowski vacuum expanding into an ancestor de Sitter space [13]. We can think of the Minkowski bubble as a point on the supersymmetric moduli space of flat vacua, in which case the moduli fields will be massive in the de Sitter region of the geometry. The Penrose diagram of this geometry is depicted in Figure 1, with the three regions labelled. We have also called the lightlike boundary between region I and region II “surface **a**” and the lightlike boundary between region II and region III “surface **b**”. Surface **b** is the horizon of the census taker.

We see that region I is the open FRW region in which the census taker finds itself at late times, that region II is the domain wall connecting the two vacua, and that region III is the “rest of the multiverse”. Σ has the topology of a two-sphere, and it can be thought of as the late-time sky of the census taker.

This geometry has a large degree of symmetry, which manifests itself differently in the three regions. In region two we can choose coordinates of the form

$$ds^2 = a(X)^2 (dX^2 - d\omega^2 + \cosh^2 \omega d\Omega_2^2), \quad (2.1)$$

which clearly has dS_{2+1} symmetry, that is $O(3, 1)$, acting along slices of fixed X . X runs from $-\infty$ to ∞ . This metric can be thought of as a warped product of dS_{2+1} over a line, which suggests the possibility of a dimensional reduction [14, 15] to a genuine dS_{2+1} theory. The future boundary of this lower dimensional theory is an S^2 which can again be thought of as Σ , and indeed it will be very convenient below for us to think of FRW/CFT as the theory at future infinity of this dS_{2+1} .

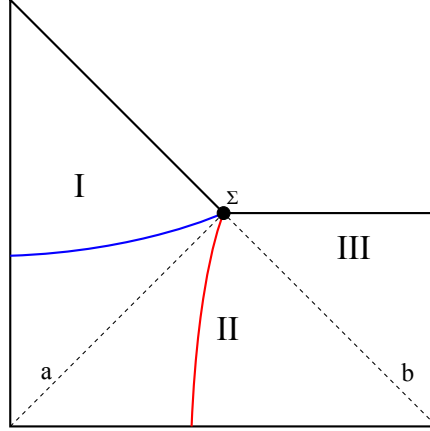


Figure 1: The various regions of the Lorentzian Coleman De Luccia geometry for the decay of dS space to $\Lambda = 0$. They are separated by the dashed lines **a** and **b**, and the colored lines are representative slices that are preserved by the symmetry. So the blue line is an FRW slice in region I and the red line is a dS_3 slice of the domain wall. The census taker can be taken as the timelike geodesic on the left side of the diagram. The geometry has a lower piece that is the time reverse of the upper half, but only this upper half is relevant for tunnelling.

Similarly in region I and region III we have coordinates of the form

$$ds^2 = a_{I,III}(T)^2 (-dT^2 + d\mathcal{H}_3^2), \quad (2.2)$$

where \mathcal{H}_3 is the hyperbolic three-plane

$$d\mathcal{H}_3^2 = dR^2 + \sinh^2 R \, d\Omega_2^2, \quad (2.3)$$

and T runs from $-\infty$ to ∞ in region I and from $-\infty$ to 0 in region III. Both regions again have $O(3,1)$ symmetry; in region I this is the FRW symmetry of the census taker. Among other things it suggests at least perturbatively that different census takers in region I should see similar physics. In fact we will see in section 6 that nonperturbative effects break this symmetry, but it is still a very useful organizing principle. The symmetry orbits near Σ are shown in figure 2. They all converge at Σ .

In region III the scale factor at late times ($T \rightarrow 0^-$) behaves as

$$a_{III}(T) \sim -\frac{1}{HT}, \quad (2.4)$$

where H is the hubble constant in the ancestor dS_{3+1} vacuum.

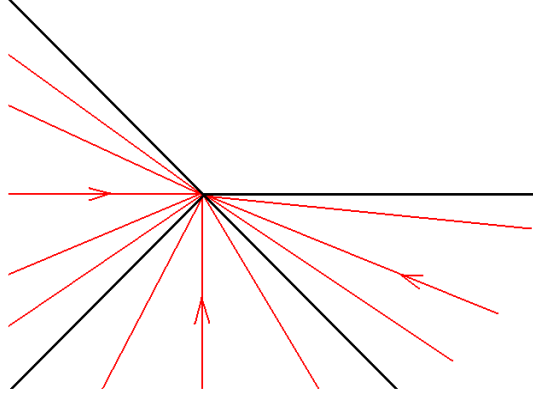


Figure 2: Orbits of $O(3, 1)$.

In region I the scale factor has the asymptotic form

$$a_I(T) \rightarrow \frac{1}{H} e^T \quad (2.5)$$

for $T \rightarrow \pm\infty$. To keep the discussion as simple as possible we will sometimes use the thin-wall approximation and set $a_I(T)$ equal to $\frac{1}{H} e^T$

We emphasize that in general Σ has a number of meanings:

- First of all Σ is the spatial boundary of region I. It is the location of the holographic degrees of freedom of the census taker's FRW region.
- Σ is the spacelike future boundary of region II, or what we called dS_{2+1} .
- Finally Σ is the boundary of region III—the-rest-of-the-multiverse, beyond the census taker's horizon.

2.2 New Coordinates

We are mainly interested in the immediate vicinity of the surface Σ . There is of course an infinite volume of space-time in all three regions in the vicinity of any point of Σ , so we really don't lose anything by zooming in on such a point. From the FRW point of view we are looking at the asymptotic behavior of the geometry out along a particular direction. For this purpose we introduce light-cone coordinates in region I,

$$T^\pm = T \pm R, \quad (2.6)$$

and take the limit of large R . We also introduce two flat tangent space coordinates, Y^1, Y^2 to describe the neighborhood of a point on the two sphere:

$$d\Omega_2^2 \rightarrow 4dY^i dY^i \quad (2.7)$$

The limiting FRW metric 2.2 satisfies

$$H^2 ds^2 = -e^{(T^+ + T^-)} dT^+ dT^- + e^{2T^+} dY^i dY^i \quad (2.8)$$

In order to attach the FRW patch to regions II and III it is helpful to change variables,

$$\begin{aligned} U^+ &= -e^{-T^+} \\ U^- &= e^{T^-} \end{aligned} \quad (2.9)$$

yielding the metric

$$H^2 ds^2 = \left(\frac{1}{U^+} \right)^2 [-dU^+ dU^- + dY^i dY^i] \quad (2.10)$$

The range of U corresponding to region I is

$$\begin{aligned} U^+ &< 0 \\ U^- &> 0 \end{aligned} \quad (2.11)$$

The metric 2.10 can be smoothly attached to the bulk 4-dimensional de Sitter space (regions II and III). The metric for de Sitter space in conformal coordinates satisfies

$$H^2 ds^2 = \left(\frac{1}{U^0} \right)^2 [-dU^+ dU^- + dY^i dY^i] \quad (2.12)$$

where $U^0 = (U^+ + U^-)/2$.

The metrics 2.10 and 2.12 agree along the surface $U^- = 0$. Thus, to sew the metrics together we extend U^- to negative values. In region $U^- < 0$ the range of variation of the coordinates is $U^+ + U^- < 0$.

The procedure outlined above corresponds to a light-like domain wall at $U^- = 0$. The generalization to a time-like domain wall is simple.

$$\begin{aligned} H^2 ds^2 &= \frac{-dU^+ dU^- + dY^2}{(U^0)^2} & (U^- < qU^+) \\ H^2 ds^2 &= \frac{-dU^+ dU^- + dY^2}{4(1+q)^2 (U^+)^2} & (U^- > qU^+) \end{aligned} \quad (2.13)$$

where the domain wall is given by $U^- = qU^+$. The parameter q is positive and less than one.

The three regions I, II, and III, are given by:

$$\begin{array}{lll} U^+ < 0, & U^- > 0 & \text{Region I} \\ U^+ < 0, & U^- < 0 & \text{Region II} \\ U^+ > 0, & U^0 < 0 & \text{Region III} \end{array} \quad (2.14)$$

Regions I and II are clearly visible to the census taker while region III is behind the horizon.

Note that in this formulation, the orbits of $O(3, 1)$ are planes of the form

$$\frac{U^+}{U^-} = c$$

with c being a constant. The census taker's horizon separating regions II and III is defined by the surface,

$$U^+ = 0 \quad (2.15)$$

None of the above depends in any important way on the thin wall approximation. There is a simple generalization that applies to all Coleman De Luccia geometries beyond the thin-wall limit. The general form involves a single function $F\left(\frac{U^+}{U^-}\right)$

$$H^2 ds^2 = F\left(\frac{U^+}{U^-}\right) \left[\frac{-dU^+ dU^- + dY^2}{(U^+)^2} \right] \quad (2.16)$$

The important feature of this metric is its scale invariance under the rescaling of all coordinates,

$$\begin{array}{ll} U^\pm & \rightarrow \lambda U^\pm \\ Y^i & \rightarrow \lambda Y^i \end{array} \quad (2.17)$$

and the $E(2)$ symmetry under translations and rotations of Y . In addition there are other transformations, which together fill out the conformal transformations on Σ . What we are really doing by focusing on the vicinity of a point is the analog of going to Poincare coordinates in anti de Sitter space.

Note that the domain wall, being an orbit of $O(3, 1)$, corresponds to a plane at constant $\frac{U^+}{U^-}$.

3 The Perturbative Fixed Point

3.1 Initial Conditions

The initial conditions⁸ for the FRW universe in region I are naturally formulated on the hypersurface $a(T) = 0$ shown as the red line in Figure 3. From equations 2.2, 2.3, and 2.5 one sees that this hypersurface is given by $T^- = -\infty$ or equivalently $U^- = 0$. Points on

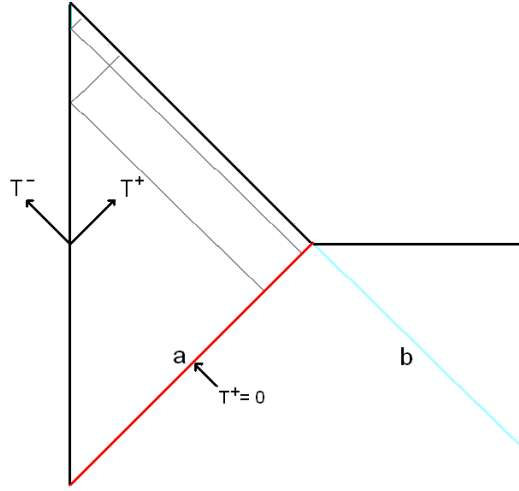


Figure 3: Initial conditions shown in red. Radiation entering at T^+ is deposited on the hat at T^- .

the initial condition surface have coordinates T^+, Y^i . Classically, initial conditions can be formulated in terms of the values of various fields $\phi(T^+, Y^i)$ on $T^- = -\infty$. These fields propagate into the FRW region and eventually deposit their information on the hat at $T^+ = \infty$. Quantum mechanically, the initial data can be formulated in terms of correlation functions of fields on the surface **a**. An example would be the two point function of a scalar field, evaluated on **a** :

$$\langle \chi(T_1^+, Y_1^i) \chi(T_2^+, Y_2^i) \rangle \quad (3.1)$$

The perturbative study of such correlation functions was undertaken in [8] [9] [11] where the emphasis was on extrapolation to Σ from region I. We will briefly state the

⁸Initial conditions for the FRW patch should be distinguished from global initial conditions for the multiverse.

results of these papers for the case of the two-point function of a minimally coupled scalar field. In the leading semiclassical approximation the 2-point function 3.1 in region I is a sum of three terms, G_1 , G_2 and G_{flat}

These terms were explained in [9] in equations (5.14), (5.20), and (5.21). Only one term, G_2 survives the limit $T^- \rightarrow -\infty$. It has the form of a sum over CFT correlation functions,

$$G_2 = \sum_{\Delta} \tilde{G}_{\Delta} e^{-\Delta(T_1^+ + T_2^+)} (1 - \cos \alpha)^{-\Delta} \quad (3.2)$$

where the Δ are a set of scaling dimensions which are either real or which come in complex conjugate pairs. The real parts of the Δ are all positive. The \tilde{G}_{Δ} are a set of numerical coefficients, and α is the angular distance between points 1 and 2 on Ω_2 . Adapting the formula to the coordinates U^{\pm}, Y^i ,

$$G_2 = \sum_{\Delta} \tilde{G}_{\Delta} |U_1^+|^{\Delta} |U_2^+|^{\Delta} |Y_1 - Y_2|^{-\Delta} \quad (3.3)$$

Each contribution has the form of a scalar-scalar correlation function in a 2-dimensional CFT. The factors $|U_1^+|^{\Delta} = e^{T^+}$ are interpreted as the wavefunction renormalization factors with U^+ being the renormalization length scale. Obviously the RG-flow toward the ultra-violet is represented by the evolution $T^+ \rightarrow \infty$. The set of scaling dimensions characterize the fixed point on Σ .

The fields on \mathbf{a} are the initial conditions for radiation entering the FRW region. The overwhelming majority of the emitted photons (and of the information that they carry) comes in at asymptotically large T^+ . It is therefore controlled by the behavior of the fixed point. Obviously, the semiclassical initial condition and the resulting radiation is sensitive to the properties of the ancestor vacuum and the domain wall, but little else. In section 6 we will show that the situation is very different when non-perturbative effects are included.

3.2 Dimensional Reduction in Region II

Consider the geometry of any surface of fixed U^+/U^- . Such surfaces are orbits of $O(3, 1)$ and are therefore homogeneous spaces. In regions I and III, where the orbits are space-like, the fixed U^+/U^- surfaces are Euclidean hyperbolic planes. In region II the orbits are time-like and are 2 + 1-dimensional de Sitter spaces. In this region the ratio U^+/U^- defines a compact space-like coordinate. The geometry has the form of a warped product of 2 + 1-dimensional de Sitter space and a line interval. By the change of variables

$$u_0 = -\sqrt{U^+ U^-}$$

$$e^{2X} = \frac{U^-}{U^+} \quad (3.4)$$

the metric 2.16 in region II can be brought to the form

$$H^2 ds^2 = a(X)^2 \left(\frac{-du_0^2 + dY^2}{u_0^2} + dX^2 \right) \quad (3.5)$$

which manifestly has the form of a warped product of $(2+1)$ -dimensional de Sitter space (in flat slicing and conformal coordinates) and the X -line. Although X runs from $-\infty$ to $+\infty$ the X -line is of finite length since $a(X)$ tends to $e^{-|X|}$ at $X = \pm\infty$.

Note that by defining

$$u_0 = -e^{-\omega} \quad (3.6)$$

the metric 3.5 can be put in the familiar exponentially expanding form

$$H^2 ds^2 = a(X)^2 (-d\omega^2 + e^{2\omega} dY^2 + dX^2). \quad (3.7)$$

For future reference we note that

$$\omega \pm X = T^\pm \quad (3.8)$$

and the metric 3.7 can be written in the form,

$$H^2 ds^2 = a(X)^2 \left(-dT^+ dT^- + e^{(T^+ + T^-)} dY^2 \right). \quad (3.9)$$

Returning to the metric as written in 3.7, the radius of curvature of each leaf of the foliation (each value of X) is given by $\frac{a(X)}{H}$ and therefore vanishes at the endpoints. This formally gives rise to a continuum of Kaluza Klein modes labeled by a continuous momentum-like quantum number k . The Kaluza Klein mass (in units of H) is given by

$$\mu^2 = 1 + k^2 \quad (3.10)$$

The geometry in 3.5 and 3.7 will be called dS_{2+1} . Because the X -interval is compact the graviton wavefunction is normalizable which implies that there is a genuine $(2+1)$ -dimensional dynamical gravitational degree of freedom [15] in dS_{2+1} . The domain wall of the Coleman De Luccia geometry is a surface of constant X . One can roughly think of dS_{2+1} as being the geometry of a thick domain wall and the $(2+1)$ dimensional gravitational field as being bound to the domain wall.

A causal patch of dS_{2+1} is defined as the interior of the backward lightcone of a point on Σ . Since dS_{2+1} is a dynamical de Sitter space its causal patches have entropy. The entropy given by standard compactification arguments is

$$S_{2+1} = \frac{m_p^2}{H^2} \int a(X)^2 dX \quad (3.11)$$

where m_p is the four-dimensional Planck mass.

Causal patches of dS_{2+1} also have a temperature which is generally of order H . Thermal radiation emitted from the census taker's horizon (surface **b** in Figure 1) is seen as radiation entering the FRW region, eventually being deposited on the hat.

The most interesting thing about dS_{2+1} from the present perspective is that its future boundary is Σ . Thus one may speculate that there is a second duality involving Σ ; namely a $(2+1)$ -dimensional dS/CFT duality.

There is one feature of dS_{2+1} that makes it conceptually different from ordinary stable de Sitter space. The far future of de Sitter space consists of many causally disconnected causal patches, which cannot communicate either with each other, or with any third party. Moreover, from experience with black holes we have learned that degrees of freedom on the two sides of a horizon are not independent.

For these reasons one may seriously doubt whether there is operational meaning of a global quantum description of de Sitter space. For example products of fields at separated comoving points cannot be measured by any physical observer.

One answer to this objection is that if de Sitter space eventually decays to flat space by reheating, then the entire space becomes visible and large scale correlations are measurable. This may be true in the context of conventional inflation, but it is irrelevant to eternal inflation, which by definition never globally reheats.

The lower dimensional dS_{2+1} is very different in this respect. Although observers who remain in region II fall out of causal contact with one another, and therefore cannot communicate, they can all be seen by the census taker. Correlations between distant points in dS_{2+1} do have operational meaning, as would a wavefunction. One could say that the Census Taker is the “Meta-observer” that overlooks the entire global dS_{2+1} .

3.3 Correlations in Region II

The FRW/CFT correspondence of [8] and [9] is one of three different limits, all converging on Σ from the three regions, I, II, and III. All three are interesting:

- Region I contains a hat and hats are the only places in which an infinite amount of information can be stored and observed.
- Region II extends back into the causal past of the hat. It contains the entire history of the census taker's causal patch, possibly including multiple bubble nucleations. We are undoubtedly in the causal past of some hat, and therefore in a region II.
- As for region III, ultimately we either want a description of the rest-of-the-multiverse or to know why one is not possible.

Thus it is extremely interesting that these three regions come together and share the same boundary, namely Σ . One would like to understand as much as possible about the theory living on Σ from the three different complementary perspectives. Earlier work concentrating on region I was largely perturbative. In this section we extend the perturbative theory to region II, mostly as a warmup for the non-perturbative analysis in Section 8. The geometry of region II is dS_{2+1} ; it is natural to suppose that it is described by some version of dS/CFT [16], [17]. To explore and confirm this possibility we will analyze how correlation functions behave as their arguments tend to Σ . Consider a $(3+1)$ -dimensional minimally coupled scalar field χ propagating in region II. In region I χ may be massless, but without supersymmetry to protect it, χ will be massive in region II. More precisely χ will have an X -dependent mass $m(X)$.

Late-time correlation functions in $(d+1)$ -dimensional de Sitter space have a form which resembles those of a Euclidean d -dimensional CFT [16]. We expect that the same is true for dS_{2+1} in the limit that the arguments approach Σ . The method of calculation in region II can be borrowed directly from [8] and was briefly described in [7].

In $(d+1)$ -dimensional dS/CFT the spectrum of operator dimensions Δ is discrete and connected with the masses of the bulk fields. For minimally-coupled-scalar fields the scaling dimensions are given in terms of the mass by

$$\Delta = \frac{d}{2} - \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}. \quad (3.12)$$

The Δ are real for sufficiently small mass, but for large mass they become complex with a real part equal to 1.

For dS_{2+1} 3.12 becomes

$$\Delta = 1 - \sqrt{1 - \frac{\mu^2}{H^2}}. \quad (3.13)$$

where μ is the appropriate mass given by 3.10. To calculate the correlation functions in dS_{2+1} we expand the four dimensional fields in a Kaluza Klein expansion. Consider for example, a massless minimally coupled field in the ancestor vacuum. The wave equation for the KK modes $u(X)$ has the form of a one-dimensional time-independent Schrodinger equation,

$$[-\partial_X^2 + V(x)]u(X) = k^2 u(x) \quad (3.14)$$

where the potential V is given by

$$V(X) = \frac{a''(X)}{a(X)} - 1. \quad (3.15)$$

Since $a(X) \rightarrow e^{-|X|}$ the potential goes rapidly to zero at large $|X|$. It is also easily seen to be attractive with a single normalizable bound state with $k^2 = -1$. This corresponds to a $(2+1)$ -dimensional mass $\mu^2 = 0$.

If the $(3+1)$ -dimensional mass of the scalar is not zero in the ancestor vacuum, the potential changes by amount $\delta V = m^2 a(X)^2$ and the eigenvalue shifts from $k^2 = -1$. For small values of m ($m < H$) the eigenvalue remains real and satisfies

$$k = \pm i(1 - \mu^2) \quad (3.16)$$

where μ^2 is of order $\frac{m^2}{H^2}$. We will see that this corresponds to an operator dimension of magnitude μ^2 at the fixed point.

All the other modes are in the continuum and oscillate. The corresponding dimensions are

$$\Delta = 1 \pm ik. \quad (3.17)$$

Complex dimensions are not surprising in the present context [16] but a continuum of dimensions is disturbing, and not what we expect from a CFT. However, this situation is familiar from [8] and has an elegant resolution. The integrals over k that define correlation are contour integrals that can be deformed so that the correlation functions become discrete sums of terms. We will repeat the argument shortly.

For the moment let us consider the bound and continuum eigenvectors of $[-\partial_X^2 + V(x)]$. The eigenvectors consist of the bound state wavefunction

$$u_B(X)$$

and a set of incoming scattering states $u_l(x)$ and $u_r(x)$ corresponding to waves coming in from the left ($x \ll 0$) and right ($x \gg 0$.) Asymptotically these waves have the form

$$u_l(k, X) = e^{ikx} + R_l(k)e^{-ikx} \quad (x \ll 0)$$

$$u_l(k, x) = T_l(k)e^{ikx} \quad (x \gg 0) \quad (3.18)$$

$$u_r(k, X) = e^{-ikx} + R_r(k)e^{ikx} \quad (x \gg 0)$$

$$u_r(k, x) = T_r(k)e^{-ikx} \quad (x \ll 0) \quad (3.19)$$

where in both cases k is positive.

One can easily prove the following:

The wave functions $u_l(k, X)$ and $u_r(k, X)$ are analytic functions of k . On the real axis the continuation to negative k is given by

$$u_{l,r}(-k, X) = u_{l,r}^*(k) \quad (3.20)$$

It follows that

$$\begin{aligned} R_{l,r}(-k) &= R_{l,r}^*(k) \\ T_{l,r}(-k) &= T_{l,r}^*(k) \end{aligned} \quad (3.21)$$

$$\begin{aligned} T_r(k) &= T_l(k) \\ R_r(k) &= -R_l(k) \frac{T(k)}{T^*(k)} \end{aligned} \quad (3.22)$$

More generally for complex k ,

$$\begin{aligned} T_r(k) &= T_l(k) \\ R_r(k) &= -R_l(k) \frac{T(k)}{T(-k)} \end{aligned} \quad (3.23)$$

The behavior of the wave functions is more complex for finite x but fortunately we know a good deal about them.

Let us consider the two-point function of the scalar field χ

$$\langle \chi(X_1, \omega_1) \chi(X_2, \omega_2) \rangle \quad (3.24)$$

in the limit $\omega_{1,2} \rightarrow \infty$.

Define α to be the angular distance on Ω_2 between the two points 1, 2, and $z = \cos \alpha$. For $\omega \rightarrow \infty$ The correlation function can be written as a sum over the eigenvectors u_B , u_l , and u_r .

$$\begin{aligned}
& \langle \chi(X_1, \omega_1) \chi(X_2, \omega_2) \rangle \\
&= \frac{1}{4\pi i a(X_1) a(X_2)} \\
& \quad \int_0^\infty \frac{dk}{\sinh \pi k} [u_l(k, X_1) u_l^*(k, X_2) + u_r(k, X_1) u_r^*(k, X_2)] e^{-(1+ik)\bar{\omega}} (1-z)^{-(1+ik)} \\
&- \frac{1}{4\pi i a(X_1) a(X_2)} \\
& \quad \int_0^\infty \frac{dk}{\sinh \pi k} [u_l(k, X_1) u_l^*(k, X_2) + u_r(k, X_1) u_r^*(k, X_2)] e^{-(1-ik)\bar{\omega}} (1-z)^{-(1-ik)} \\
&+ \text{bound state contribution.} \tag{3.25}
\end{aligned}$$

where $\bar{\omega} = \omega_1 + \omega_2$.

Now we use the fact that on the real k axis the wave functions can be analytically continued to negative k ,

$$u_{l,r}(-k, X) = u_{l,r}^*(k, X), \tag{3.26}$$

to write the continuum contribution to the correlation function as a contour integral over the real axis:

$$\begin{aligned}
& \langle \chi(X_1, \omega_1) \chi(X_2, \omega_2) \rangle \\
&= \frac{1}{8\pi i a(X_1) a(X_2)} \\
& \quad \int_{-\infty}^\infty \frac{dk}{\sinh \pi k} [u_l(k, X_1) u_l(-k, X_2) + u_r(k, X_1) u_r(-k, X_2)] e^{-(1+ik)\bar{\omega}} (1-z)^{-(1+ik)} \\
&- \frac{1}{8\pi i a(X_1) a(X_2)} \\
& \quad \int_{-\infty}^\infty \frac{dk}{\sinh \pi k} [u_l(k, X_1) u_l(-k, X_2) + u_r(k, X_1) u_r(-k, X_2)] e^{-(1-ik)\bar{\omega}} (1-z)^{-(1-ik)} \\
&+ \text{bound state contribution.} \tag{3.27}
\end{aligned}$$

In this form the correlation function is a sum over a continuum of complex conformal dimensions. This is so because the correlation function for a scalar field of scaling dimension Δ has the form

$$G_\Delta \sim (1 - z)^{-\Delta}. \quad (3.28)$$

To turn the expression in 3.27 into a discrete sum of terms we use the fact that the wave functions $u(k, X)$ are analytic functions of k except for discrete poles. The poles in the upper half plane are associated with bound states and are strictly on the imaginary k axis. In the lower half plane they occur on the imaginary axis, or they occur in pairs symmetrically located with respect to the imaginary axis. The integrands also have poles at the imaginary integers due to the factors $\sinh^{-1}(\pi k)$.

For large ω the contour for the first integral,

$$\frac{1}{8\pi i a(X_1)a(X_2)} \int \frac{dk}{\sinh \pi k} [u_l(k, X_1)u_l(-k, X_2) + u_r(k, X_1)u_r(-k, X_2)] e^{-(1+ik)\bar{\omega}} (1-z)^{-(1+ik)}, \quad (3.29)$$

can be closed in the lower half plane and one picks up a discrete set of terms of definite dimension. The other integral is closed in the upper half plane and also produces a similar set of discrete terms. Each term is accompanied by a residue function that depends on X . One may think of these as Kaluza Klein wave functions, but they do not form an orthonormal family.

Let us consider a specific contribution from the first integral. The contribution from a pole at $k = i\kappa$ is,

$$F(X_1, X_2) e^{-(1-\kappa)\bar{\omega}} \frac{1}{(1-z)^{1-\kappa}} \quad (3.30)$$

For this term $\kappa < 0$ since the contour is closed in the lower half plane. This means that the dimension $\Delta = (1 - \kappa)$ is greater than 1. There are no very low dimension correlation functions from this term.

The same is true of the other integral which gives contributions

$$F(X_1, X_2) e^{-(1+\kappa)\bar{\omega}} \frac{1}{(1-z)^{1+\kappa}} \quad (3.31)$$

but this time with $\kappa > 0$. We find the continuum contributions are of the conformal kind but with dimensions all satisfying $\Delta > 1$.

Later we will be especially interested in very low dimensional contributions. The only place that they can come from, perturbatively, is the bound state contribution. In the (unphysical) massless case the bound state is a solution of the Schrodinger equation with

eigenvalue $k^2 = -1$. In the small mass case the eigenvalue gets shifted to a value $k = \pm i(1 - \epsilon)$ where

$$\epsilon \sim \frac{m}{H} \quad (3.32)$$

In that event the bound state contribution has the form

$$\langle \chi(X_1, \omega_1) \chi(X_2, \omega_2) \rangle_{BS} = \frac{u_B(X_1)}{a(X_1)} \frac{u_B(X_2)}{a(X_2)} e^{-\epsilon(\omega_1 + \omega_2)} \frac{1}{(1 - z)^\epsilon} \quad (3.33)$$

For completely massless scalars the bound state wave function $U_B(X)$ is exactly equal to $a(X)$ and the ratios $\frac{u_B(X)}{a(X)}$ are unity. For $m \neq 0$ they do not cancel but asymptotically for large $|X|$ the ratios satisfy

$$\frac{u_B(X)}{a(X)} \rightarrow e^{\epsilon X}. \quad (3.34)$$

In other words it is the correlation function for a dimension ϵ scalar on the surface Σ . The approach to the surface is controlled by the wave function renormalization factors $e^{-\epsilon(\omega_1 + \omega_2)}$.

3.4 Back to the Initial Condition Surface

The initial conditions for the census taker's bubble is given by the values of correlation functions on the surface \mathbf{a} . That surface is defined by allowing $X \rightarrow -\infty$ and $\omega \rightarrow \infty$ with $\omega + X$ fixed, or from 3.8, with T^+ fixed.

The easiest way to take that limit is to go back to equation 3.27 and take the limit using 3.18, 3.19, and 3.22. In the limit $X \rightarrow -\infty$, $\omega \rightarrow \infty$ one finds

$$\begin{aligned} & 8\pi i \langle \chi(X_1, \omega_1) \chi(X_2, \omega_2) \rangle \\ &= e^{-X_1} e^{-X_2} \int_{-\infty}^{\infty} \frac{dk}{\sinh \pi k} \left[2 \cosh k(X_1 - X_2) + R(k) e^{ik(X_1 + X_2)} + R(-k) e^{-ik(X_1 + X_2)} \right] \\ & \quad \left\{ e^{-(1+ik)(\omega_1 + \omega_2)} (1 - z)^{-(1+ik)} - e^{-(1-ik)(\omega_1 + \omega_2)} (1 - z)^{-(1-ik)} \right\} \\ & \quad + \text{bound state contribution.} \end{aligned} \quad (3.35)$$

The factors $e^{-X_1} e^{-X_2}$ are the asymptotic values of

$$\frac{1}{a(X_1)}$$

and

$$\frac{1}{a(X_2)}$$

relating the correlation function to \hat{G} .

The bound state contribution is extremely simple.

$$\begin{aligned} 8\pi i \langle \chi(X_1, \omega_1) \chi(X_2, \omega_2) \rangle &= e^{-\epsilon(\omega_1 + X_1)} e^{-\epsilon(\omega_2 + X_2)} \frac{1}{(1-z)^\epsilon} \\ &= e^{-\epsilon T_1^+} e^{-\epsilon T_2^+} \frac{1}{(1-z)^\epsilon} \end{aligned} \quad (3.36)$$

The remaining contributions have the same form with ϵ being replaced by operator dimensions Δ . We see that the correlation function on the initial condition surface has the form in Equation 3.2.

The structure of these correlations emphasizes the deep relationship between RG flow of the $(2+1)$ -dimensional dS/CFT to the T^+ dependence of the initial condition on the surface \mathbf{a} . Since the surfaces of constant T^+ are the Census Taker's past light-cones, we see how the RG flow is intimately connected with the evolution of the census taker's observations.

4 Doubled dS/CFT in Region II

Whether or not a global CFT description of ordinary de Sitter space makes operational sense, the corresponding dS/CFT description of dS_{2+1} seems well-motivated. Since Σ is the asymptotic future of dS_{2+1} the dS/CFT would be a description of this surface. However it would not be the same limit as the FRW/CFT theory describes: the two limits approach Σ from different directions. Approaching Σ along spacelike surfaces in region I is very similar to approaching the boundary of Euclidean AdS space. In region II Σ is approached along timelike directions. There is a big difference in how observables and expectation values are defined in the two cases [18]. Nevertheless the two theories of Σ must be intimately connected.

This section is not essential to the arguments of the present paper but it is included for a couple of reasons. The first is just to show how earlier conjectures about dS/CFT can be applied to dS_{2+1} . The second is that it may suggest an eventual microscopic framework.

Applying the arguments of [17] we assume that the Wheeler DeWitt wavefunction of dS_{2+1} is given by the partition function of a Euclidean two-dimensional CFT with fields A and sources h . The sources are the usual $(3+1)$ -dimensional fields. Among the them

are the components of the spatial metric, light scalars, and other bulk fields. They also include the “multiverse fields ” [19] that we will explain later. The A on the other hand, are analogous to gauge fields and other boundary degrees of freedom in ADS/CFT duality. We will refer to them as gauge fields.

We assume the action for the gauge theory is

$$I = \int d^2Y L(A; h). \quad (4.1)$$

The Wheeler DeWitt wavefunction is a functional of the sources,

$$\Psi(h) = \int e^{\int d^2Y L(A; h)} dA \quad (4.2)$$

The gauge fields A are not the observables that we are interested in. Our interest lies in the statistics of the sources. The probability for a given configuration of sources is given by the Born rule,

$$P(h) = \Psi^*(h)\Psi(h) \quad (4.3)$$

which can be directly computed using a doubling trick,

$$P(h) = \Psi(h) = \int e^{\int d^2Y \{L(A; h) + L^*(B; h)\}} dA dB \quad (4.4)$$

where B are a second set of gauge fields. Note that the sources are the same in the A and B terms.

Now let's consider the expectation values of functionals of h . Let $F(h)$ be such a functional.

$$\langle F(h) \rangle = \int F(h) e^{\int d^2Y \{L(A; h) + L^*(B; h)\}} dA dB dh \quad (4.5)$$

In other words the expectation values of bulk fields at late time is given in terms of a *doubled* theory with dynamical fields that include two copies of the gauge fields and a single copy of the sources. The fact that we have to integrate over the sources means that the machinery for computing expectation values is a form of two-dimensional Euclidean quantum gravity. If one gauge-fixes to conformal gauge, then the theory is described as two copies of “matter” coupled to a Liouville degree of freedom.

One might expect that the Liouville field defined in this way would be the same Liouville field that appeared in the FRW/CFT theory, but that would be a mistake; the geometry of Σ is defined in two different ways which are not continuations of each other. In the dS/CFT description of dS_{2+1} the gravitational field is an average over the X coordinate.

The continuation of X into region I is FRW time. The Liouville field in FRW/CFT is not an average over time; it is time.

From another more technical point of view the two Liouville theories cannot be the same. It is known that the logarithm of the Wheeler DeWitt wave functional does have a Liouville contribution in $(2+1)$ dimensions [20]. As in the FRW/CFT case the magnitude of the central charge is of order the ancestor entropy. However, in the dS_{2+1} case the central charge is imaginary whereas in the FRW case it is real and negative.

In fact the difference between the theories becomes even clearer in the doubled theory. In that case the central charge of the dS/CFT Liouville theory is zero, since it cancels when computing $\Psi^*\Psi$. By contrast the large negative central charge of the FRW/CFT puts the theory into the deep semiclassical regime with a very stable spherical saddle point. It is evident that in order to stabilize the spatial geometry in $(2+1)$ -dimensional de Sitter space one must fix a time variable in some way. Despite the fact that the Liouville fields are different in the dS/CFT and FRW/CFT descriptions of Σ there must be a close connection between the theories.

It is apparent that the limit $u^0 \rightarrow 0$ defines the UV fixed point behavior of the doubled theory in much the same way that $U^+ \rightarrow 0$ defined the fixed point of the FRW/CFT theory. In fact we expect the operator dimensions to be the same.

The doubled theory is a machine for calculating correlations in dS_{2+1} . Some approximation to it should be able to reproduce the correlation functions of the last section including the low dimension operators associated with very light scalars. Such low dimension correlations fall off slowly and obviously control the largest scale features. One might think there are not very many low dimension operators. However this is not so. In the next section we will explain how non-perturbative effects create an enormous discretuum of extremely low dimension fields on Σ , which essentially describe the entire landscape of metastable vacua. These low dimensional fields may provide a rigorous classification of the landscape.

The existence of low dimension scalars means that the fixed point of the doubled CFT is unstable with respect to flowing toward the infrared. Even in the perturbative case, if the mass of the scalar field χ is smaller than H there is a relevant operator in the doubled CFT. How do we understand these instabilities⁹?

⁹Whether to think of the UV or the IR as the initial state of an RG flow is ambiguous. In condensed matter physics one usually thinks of the starting point, or initial condition, being the UV end, the output being the IR behavior. In cosmology the IR corresponds to the initial condition and the UV to the late-time behavior.

The UV limit in region II is late time, i.e., $\omega \rightarrow \infty$. Flowing toward the infrared means running toward early time. As one runs toward early time, relevant perturbations will eventually become strong. This means that things become sensitive to global initial conditions¹⁰ [21]. The closer the UV is to the fixed point, the longer (into the past) it will take to feel the relevant operators. To put it another way, very small relevant perturbations mean the initial condition is far in the past, while large relevant perturbations mean that the bubble nucleated close to the initial condition surface.

5 The Non-Perturbative Fixed Point

5.1 Non-Perturbative Effects

Describing eternal inflation in terms of the hat immediately raises the question of which hat? A Coleman De Luccia instanton describes tunneling from a particular ancestor to a particular final flat vacuum. Are there a large class of different descriptions for each possible initial and final state? Or are they all the same? The problem is exacerbated by two facts. The first is that the number of classically stable de Sitter vacua is likely to be extremely large. The second is that they may all be subject to non-perturbative instabilities and therefore not be accurately defined.

The answer that we will propose is that the FRW/CFT duality is not about any specific transition. Mobility on the landscape, due to non-perturbative effects, cause an RG flow to a common final UV fixed point that contains information about the entire landscape.

The non-perturbative effect that drive this RG flow is the nucleation of Coleman De Luccia bubbles. Such bubbles can collide with the Census Taker's bubble and perturb the initial condition on the surface \mathbf{a} . To the census taker these collisions appear as disc-like disturbances concentrated over a small region of the Y coordinates. The later the colliding bubble nucleates the smaller the disturbed region. From the point of view of the FRW/CFT theory living on Σ this type of configuration might be thought of as a 2-dimensional Euclidean instanton.

There are several types of bubble collisions to consider:

- Collision with another FRW bubble on the same moduli space: This is shown in Figure 4. No domain wall is created by the collision although some energy is released and radiated into the FRW regions, and the two bubbles merge to form a single hat.

¹⁰We will assume that the global initial condition consists of specifying an initial de Sitter vacuum with no bubbles on an initial global spacelike hypersurface.

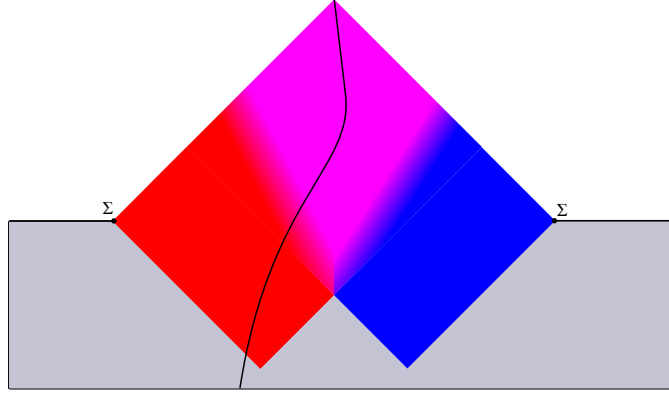


Figure 4: A slice of the collision of two $\Lambda = 0$ bubbles on the same moduli space. A census taker that enters from either side ends up in the same hat. The two indicated points are part of the boundary two-sphere Σ .

The vacuum in the colliding bubble may or may not be at the same point in the moduli space as the original bubble. If it is not, there is still no domain wall but the two vacua will bleed into each other. What this means is that such collisions will contribute to the fluctuations of the massless moduli fields in the FRW region, and also on Σ . Note in Figure 4 that some part of what would have been in region II if only the red bubble nucleated now becomes part of region I; we can think of collisions with other bubbles on the same moduli space as “pushing out” Σ over some angular region. This is much like the displacement of a black hole horizon by an in-falling shell of matter.

- Collision with a de Sitter bubble: This is shown schematically in Figure 5.

The collision results in a domain wall, shown as dark blue, between the census taker’s bubble and the de Sitter bubble. It neither destroys the hat or Σ but it obviously changes the initial conditions for large T^+ or equivalently small $|U^+|$. This can be thought of as “pulling in” Σ over an angular region.

The future boundary of the colliding bubble is an inflating boundary similar to that of the ancestor.

- Collision with a flat bubble on a disconnected moduli space. If, as we expect, there are disconnected moduli spaces (for example vacua with different supersymmetry) then a collision between them will produce a domain wall. The geometry of such a collision will tend to the Vilenkin-IPser-Sikivie geometry of [22]. The domain wall

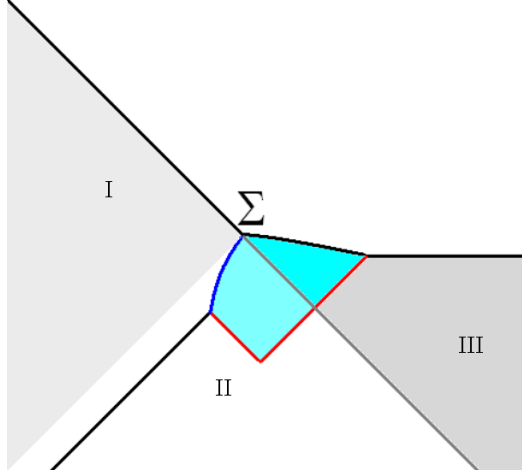


Figure 5: Collision with a de Sitter bubble. The light grey and dark grey regions are in I and III respectively. The white region is in I.

will accelerate away from both regions. The intrinsic geometry of the domain wall will be locally de Sitter and will have an acceleration temperature. From the point of view of a census taker on one side or the other, this type of collision is very similar to a collision with a de Sitter bubble. Unlike the case where both bubbles are on the same moduli space, there are now two hats produced as shown in figure 6.

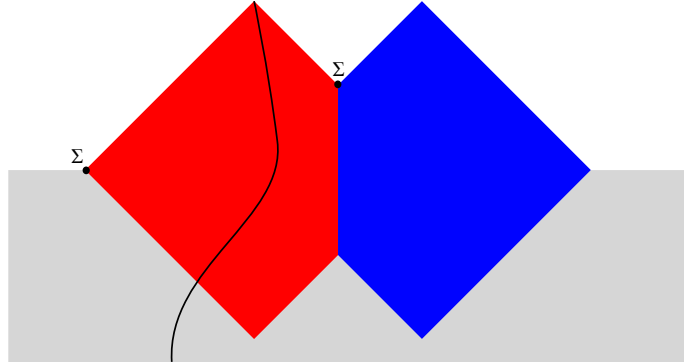


Figure 6: Collision of two $\Lambda = 0$ bubbles which are on different moduli spaces. The labelled points are part of Σ for the drawn census taker.

- Collision with a crunching vacuum of negative cosmological constant which could be

a supersymmetric ADS type vacuum: This is the most dangerous and most poorly understood case. In general it will lead to a domain wall. The tension of the domain wall is bounded from below by a BPS bound. There are two cases [12]. In the first case the tension is greater than the BPS bound, and the domain wall accelerates away from the Census Taker's bubble. The picture is similar to Figure 4 except that the future boundary of the bubble is a singularity instead of an inflating region. An important fact is that for this case the domain wall is eternally accelerating and therefore remains hot during its entire infinite history. This means that it is not dead. It is the source of new nucleation events.

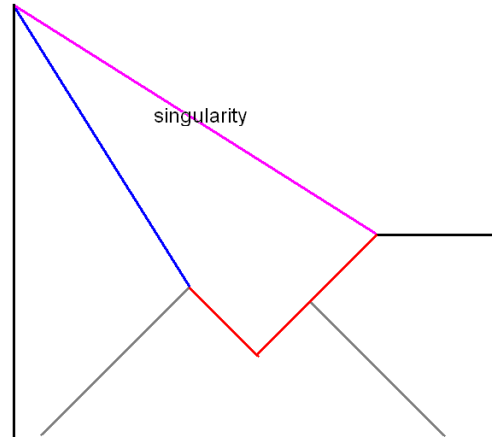


Figure 7: Saturating the BPS domain wall bound. The domain wall appears to push Σ all the way up to the top of the hat, but remember that this happens only over a small angular region.

- Alternatively if the domain wall between the crunch and the census taker's bubble saturates the BPS bound then it does not accelerate. It penetrates the census taker's bubble and eventually hits time-like infinity. This is shown in Figure 7. The conformal diagram makes the collision look much more dangerous to the census taker than it really is. For one thing the perturbation only occupies a small angle in the census taker's sky, and for another, the tip of the hat is really an infinite region, most of which is far from the census taker.

In sections 5.3-6 we will discuss how these nonperturbative effects affect FRW/CFT, but we first build up some intuition from a different source.

5.2 Some Lessons from Trees

There is evidence that perturbative correlation functions in de Sitter space have conformal limits on the future boundary. Non-perturbatively, when transitions between de Sitter vacua are included, the evidence is more limited and only applies if there are no terminal vacua. Terminal vacua destroy conformal invariance but leave scale-invariance unbroken. In this section we will review the results of [19] regarding these issues. For the moment the discussion is general and includes the cases of $(3+1)$ -dimensional de Sitter space and also dS_{2+1} .

We begin with a landscape consisting of de Sitter vacua, assuming for the moment that there are no terminal vacua with either vanishing or negative vacuum energy. The de Sitter vacua are labeled by a “color” label n . To describe the population of colors a cutoff time must be introduced. Call the cutoff time t . For the moment the precise definition of time will not be specified. Bubbles are allowed to nucleate before the cutoff, and then grow and populate a portion of future infinity. The fraction of comoving volume of future infinity populated by color n is called $P(n)$.

To regulate the bubble population ignore all bubbles whose nucleation-centers occur later than t . A bubble that nucleates earlier than t leaves a colored ball on future infinity. One can ask how much of the coordinate volume on future infinity is covered by color n when the cutoff is t . The answer is governed by rate equations that were analyzed in [23][19]. We will closely follow the arguments of [19] with slight changes of notation.

One begins with a set of rate equations [23] describing the population as a function of cutoff time t ,

$$\frac{dP_n}{dt} = \dot{P}_n = \sum_m (\gamma_{nm} P_m - \gamma_{mn} P_n) \quad (5.1)$$

where γ_{nm} is the dimensionless transition rate for a bubble of type n to nucleate in a vacuum of type m . The equation can be written,

$$\frac{dP_n}{dt} = G_{mn} P_m \quad (5.2)$$

If one adds the ingredient of detailed balance the transition rates satisfy

$$\gamma_{nm} = e^{S_n} M_{nm} \quad (5.3)$$

where M_{nm} is a symmetric matrix. In this equation S_n represents the de Sitter entropy of the n^{th} vacuum.

Making the change of variables

$$\Phi_n = e^{-S_n} P_n \quad (5.4)$$

one finds the equation

$$\dot{\Phi}_n = \sum_m (M_{nm} e^{\frac{S_m + S_n}{2}} \Phi_m - M_{mn} e^{S_m} \Phi_n). \quad (5.5)$$

When written out in matrix notation the equation has the form

$$\dot{\Phi}_n = \sum_m T_{nm} \Phi_m \quad (5.6)$$

where T_{nm} is symmetric. Therefore there are a complete set of orthogonal eigenvectors $\Phi_I(m)$ of T which evolve by simple rescaling,

$$\Phi_I(m, t) = e^{-\Delta_I t} \Phi_I(m, 0) \quad (5.7)$$

where $e^{-\Delta_I}$ is the I^{th} eigenvalue of T_{nm} .

The Δ are all positive except for the leading eigenvector which has $\Delta = 0$. This means that a generic initial condition will flow to a stationary probability distribution, and by inspecting the equations one can see that the stationary probabilities P_n satisfy

$$P_n = \frac{1}{Z} e^{S_n} \quad (5.8)$$

where Z is a normalization factor.

This stationary probability distribution does not represent any individual local minimum on the landscape. Rather it represents a universal bath of bubbles at asymptotic time. The distribution as the cutoff tends to infinity becomes independent of the initial conditions, and not surprisingly is scale invariant. But the evidence from both [24] and the tree-like model [19] is that if detailed balance is satisfied then a stronger symmetry prevails, namely that the asymptotic population is invariant with respect to conformal transformations of future infinity.

In [19] it was shown that there is a vast discretuum of conformal *multiverse fields* that are associated with the eigenvectors Φ_I . One can identify a multiverse field with each color n . The value of each field is either 0 or 1 depending on whether a point on the future boundary is that color or not. Multiverse fields have not played a role in the dS/CFT context although they certainly should. The sources, or what is the same thing,

the boundary conditions on the CFT partition function in 4.4 must specify which color vacuum occupies each region of space.

One may choose the basis of multiverse fields to be labeled by color, or alternatively, superpositions with definite scaling dimensions can be defined by the eigenvectors Φ_I . The scaling dimension of Φ_I is Δ_I , and there are of course a great many of them if the Landscape is rich.

The values of the Δ are determined by the rate equations and are generally of order the transition rates γ . Assuming the existence of a great many metastable long lived vacua, there will also be an equally large number of very low dimension multiverse fields. The stationary eigenvector with dimension zero, 5.8, is identified with the unit operator in the sense of a CFT. All the other dimensions are positive.

The introduction of terminal vacua significantly modify these conclusions, and in particular, destroy conformal symmetry. We will continue to use the color notation n for de Sitter vacua, and collectively lump the terminal vacua into the “the terminal.” The effect of the terminal is to introduce an additional term into the rate equations,

$$\frac{dP_n}{dt} = \sum_m (\gamma_{nm}P_m - \gamma_{mn}P_n) - \gamma_n P_n \quad (5.9)$$

where γ_n is the total rate to decay from color n to the terminal. It is obvious from these equations that all the P_m tend to zero at late time. The loss of probability is of course exactly compensated by the increasing probability P_t for the terminal.

Quantitatively, the eigenvalues of 5.9 are all less than 1 and the scaling dimensions Δ are all positive. The smallest of them, Δ_D , defines the *dominant* eigenvector. The dominant eigenvector replaces the stationary eigenvector. In addition there is a discretuum of very small sub-leading scaling dimensions Δ_I , all satisfying $\Delta_I > \Delta_D$.

Eventually the terminal probability tends to unity and all others to zero. The fixed point is replaced by

$$\begin{aligned} P_m &= 0 \\ P_t &= 1 \end{aligned} \quad (5.10)$$

However, the situation is not as dull as it seems. Garriga and Vilenkin [25] have speculated that there may be a “conformal field theory on a fractal” living on the inflating fractal that remains on future infinity after the terminal is removed. In [19] it was shown that an interesting structure survives in the tree-like model, although there is no evidence that it is a field theory. The structure is defined by asking conditional questions of the following kind:

Given a point p , what is the probability that it has color n given that it has not been swallowed up by a terminal bubble?

Similarly, Given two points p_1 and p_2 , what is the conditional probability that they have color n_1 and n_2 , given that neither has been swallowed up by a terminal bubble?:

Such questions define a collection of conditional correlation functions

$$\begin{aligned}
C_n(p) \\
C_{n_1, n_2}(p_1, p_2) \\
C_{n_1, n_2, n_3}(p_1, p_2, p_3) \\
\ldots
\end{aligned} \tag{5.11}$$

The corresponding absolute probabilities go to zero as $\exp -\Delta_D t$ but the conditional probabilities have these factors divided out.

Terminals destroy the conformal theory that lives on the future boundary but there is evidence that they leave a well defined scale-invariant structure [19]. Linear combinations of the correlators corresponding to eigenvectors of the rate equation 5.9 have definite scaling dimension, but the bigger conformal symmetry is broken. The arguments of [19] suggest that two-point correlators of these scaling fields have the form,

$$\langle \Phi_I(Y_1) \Phi_J(Y_2) \rangle = \sum_n e^{-\frac{S_n}{2}} \Phi_D(n) \Phi_I(n) \Phi_J(n) |Y_1 - Y_2|^{\Delta_D - \Delta_I - \Delta_J}. \tag{5.12}$$

The fact that these are not diagonal in dimension shows that the special conformal transformations that hold two points fixed on future infinity are no longer symmetries. The symmetries that do remain (the symmetries of the conditional correlations) are scale transformations, translations, and rotations of the future boundary.

In the absence of terminals there is no preferred frame of reference. The same symmetry that acts as conformal symmetry on future infinity acts in the bulk on the time coordinate, transforming from one frame to another. The breaking of conformal symmetry by terminals picks a preferred reference frame or time coordinate, namely the time coordinate for which the correlation functions tend to zero exponentially. It may seem surprising that terminals prejudice the choice of reference frame, but this is not a new effect. In [21] it was called *The Persistence of Memory*. The point of that paper was that one must start with an initial condition such that on some space-like surface no terminal bubbles are present. The memory of that surface is not erased by time if there are terminal vacua.

However, apart from the choice of reference frame, correlation functions in [19] were shown to be universal, scale invariant, and controlled by the dominant eigenvector of the rate equations.

These are the lessons that we will apply to dS_{2+1} .

5.3 The Fixed Point

All of the non-perturbative effects described in section 5.1 except for collisions of $\Lambda = 0$ bubbles on the same moduli space arose from non-perturbative fluctuations in region II. As we discussed in section 3.2, at least perturbatively region II has dS_{2+1} symmetry. If we for a moment neglect these same-moduli space collisions, it seems reasonable to apply the lessons of the last section to these non-perturbative processes in dS_{2+1} . dS bubbles, non-BPS AdS bubbles, and bubbles of disconnected moduli space all produce circular regions on the census taker’s sky which contain smaller fractal structure inside of them, and we will argue that they can be thought of as “dS bubbles” in the dS_{2+1} theory. BPS domain walls with crunching AdS regions on the other side produce “dead” regions, which contain no smaller structure. We will claim that these can be thought of as terminals in the dS_{2+1} theory. For clarity we will postpone further discussion of these BPS domain walls to the following section. We will also continue to ignore same-moduli space collisions for a little longer.

Begin with an FRW bubble in a red ancestor vacuum as shown in Figure 8. From the

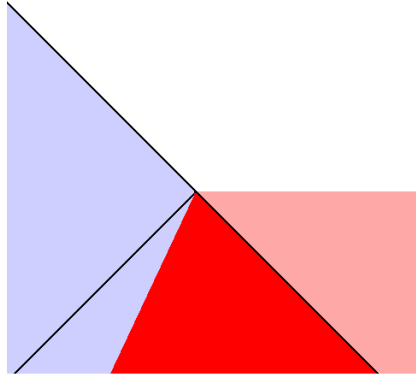


Figure 8: A FRW bubble in a red ancestor vacuum.

(2 + 1)-dimensional perspective this is just a red de Sitter space. Most of the degrees

of freedom are located near $X = 0$ (see section 4, equation 3.11). To keep track of the vacuum adjacent to the FRW region we can imagine painting Σ red.

Next consider the nucleation of a de Sitter bubble of vacuum type green, taking place in region II.¹¹ The green bubble will collide with the original FRW bubble as shown

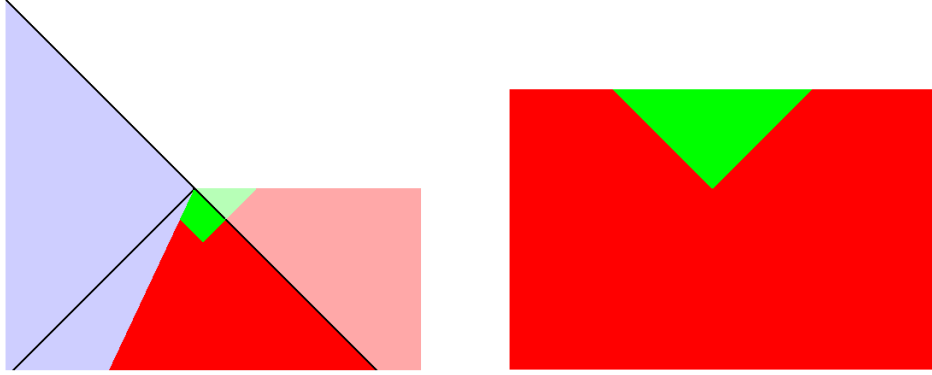


Figure 9: Collision with a green de Sitter bubble in the 3+1 geometry (left) appears as a nucleation event in the 2+1 geometry (right).

in the left panel of Figure 9. From the $(2 + 1)$ -dimensional viewpoint this is a bubble nucleation of a conventional kind, as shown in the right panel of Figure 9. The bubble will grow and eventually occupy a disc-like region on Σ . We will imagine the disc to be painted green as in Figure 10

Strictly speaking, a nucleation event is characterized by more than just the vacuum type. In addition to knowing when the nucleation event took place (ω) we also have to know where along the X axis it occurred. We may try to keep track of the location of the nucleation event by including an additional label. However, this seems awkward. One should keep in mind that a de Sitter vacuum has many hidden labels associated with the large number of microstates, all of which are lumped into a single macroscopic state. The label X is just another one of these degrees of freedom. Thus it is convenient to integrate over the X coordinate of the nucleation site. The following facts suggests that we integrate over X , keeping ω fixed.

¹¹This argument also applies to AdS bubbles which have non-BPS domain walls with the census taker bubble, as well as $\Lambda = 0$ bubbles on a disconnected moduli space. For simplicity we will continue to refer to them just as dS bubbles.

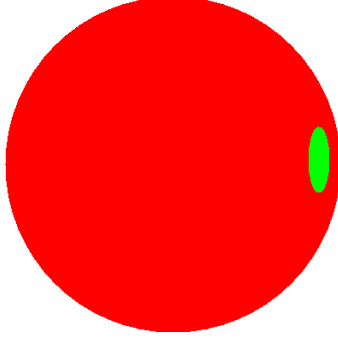


Figure 10: A de Sitter Bubble in region II blocks out a disc in Σ .

- A natural definition of the RG flow parameter describing Σ is the coordinate size of a perturbation on the two-sphere Ω_2 . Two bubbles of the same coordinate size and identical color will have similar appearance to the census taker. So it makes sense to lump them together.
- The coordinate size of a green disc on Σ is a function of where, in region II, the green bubble nucleated. In other words it is a function of ω and X . However, when the geometry of bubble collisions is examined, it is found that the coordinate size of the green disc only depends on ω . It is independent of X . This is shown in the appendix.

From the census taker's viewpoint there is very little difference between bubbles that nucleate at (ω, X) and (ω, X') . We can absorb the X coordinate into the internal degrees of freedom of the bubble by integrating over X . This allows us to organize bubbles according to a natural scale-size. Once that is done, bubble nucleation in region II can be identified with $(2 + 1)$ -dimensional bubble nucleation in dS_{2+1} , and the arguments of [19] should apply. In particular the analysis of Section 9 based on rate equations should hold. Note that the time parameter in the rate equations is replaced by ω . For example, the fractional coordinate area $P(n)$ satisfies the rate equation

$$\frac{dP_n}{d\omega} = \dot{P}_n = \sum_m (\gamma_{nm} P_m - \gamma_{mn} P_n) \quad (5.13)$$

The rest of the analysis is the same as in Section 9. In particular, in the absence of terminal $(2 + 1)$ -dimensional vacua, there is a stationary probability distribution representing a universal bath of bubbles, contiguous with Σ . As the cutoff (ω) tends to infinity the distribution becomes independent of the initial conditions and conformally invariant.

The discretuum of very low dimensional conformal *multiverse fields* that were described in [19] will be present in the spectrum of dimensions in the $(2+1)$ -dimensional dimensional theory.

Let's now consider collisions with bubbles on the same moduli space. As we saw in 4, these collisions simply merge with the hat and push it out a little bit. They can really be thought as part of the dynamics of region I. As more bubbles are nucleated at smaller scales and in different directions a fractal pattern of values of the moduli will be produced throughout region I. It is this fractal distribution that we truly identify as hat, and it is clear that for a given connected moduli space there can only be one type. In figure 11 this procdure is shown for a few collisions. It may appear that the bubble that the census taker enters first is special, but from 4 it should be clear that there is symmetry between these different colliding bubbles and the final pattern cannot depend on this. This distribution is the window through which the census taker looks down on region II.

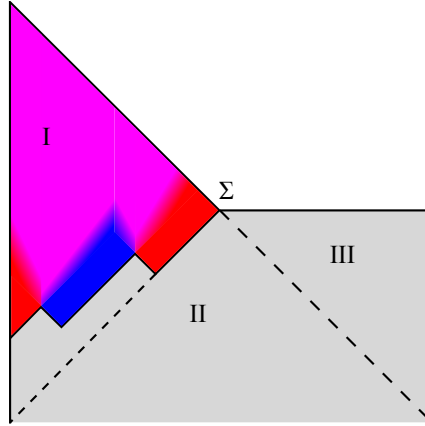


Figure 11: Same-moduli space collisions as region I dynamics.

An important issue is that we don't actually expect that the region I dynamics are decoupled from those in region II, as our discussion might have implied. Both of these dynamics affect the generation of the pattern on the census-takers sky, and there will be some coupling between them. The real proposal is that there is some big rate equation that involves both the region I dynamics and the region II dynamics, and that together it flows to a joint universal fixed point.

Whether or not the fixed point is the same for hats that correspond to *different* moduli spaces is a more difficult question. We see no reason why hats with disconnected moduli spaces need to be described by the same fixed point. A more subtle issue is when there are

different moduli spaces that are connected at infinity, as often happens in string theory where different moduli spaces are usually (always?) connected by a decompactification limit. Perhaps this connection is enough to combine the moduli spaces into a single type of hat, but more analysis is clearly required and we leave it for future work.

To summarize, we expect the theory on Σ to be characterized by three important features:

- The fixed point behavior of the FRW/CFT is universal. It does not depend on any initial condition including the ancestor vacuum. If there are disconnected moduli spaces then it seems likely to depend on which one the hat ends up in, so we probably need to condition on this.
- The landscape of de Sitter vacua is reflected in a discretuum of very low dimension multiverse operators.
- Unlike the global dS/CFT the entire field theory on Σ is visible to a single observer. That observer does not live on dS_{2+1} , but rather in the hat. Unlike the perturbative case, the radiation entering the hat from the initial condition surface carries information about the entire multiverse.

The discretuum of low dimension operators is unusual but it has an important meaning that deserves discussion. Operators of low dimension are relevant in the RG sense and imply that the fixed point is unstable as the flow proceeds toward the infrared. A large number of such relevant operators indicates that the flow may split up into a equally large number of distinct IR limits. One can turn it around and say that a great many starting points in the IR will all lead to the same UV fixed point. These IR starting points simply reflect the multiplicity of the landscape itself. The discretuum of de Sitter vacua is directly connected to the discretuum of low dimensional fields on Σ . It is possible that this provides a precise characterization of the Landscape of de Sitter vacua.

These low dimension operators also play an important role in determining leading corrections to correlators as the UV fixed point is approached. To understand this consider a general d dimensional CFT in finite volume ($\sim L^d$) perturbed by a relevant operator O_j with dimension Δ_j . The action is

$$S = S_{\text{CFT}} + u_j \int d^d z O_j(z) \quad (5.14)$$

The strength of the perturbation is set by demanding that its effect be at most order one at the infrared scale, where the FRW/CFT flow to the ultraviolet begins. So $u_j \sim$

$(1/L)^{d-\Delta_j}$. The two point function of another operator O_i can be computed perturbatively in u_i .

$$\langle O_i(x)O_i(y) \rangle = \langle O_i(x)O_i(y) \rangle + u_j \int d^d z \langle O_i(x)O_i(y)O_j(z) \rangle + \dots \quad (5.15)$$

Using the conformally invariant form for the three point function and assuming that the operator product coefficient C_{ij} is nonvanishing we find

$$\langle O_i(x)O_i(y) \rangle = \frac{1}{|x-y|^{2\Delta_i}} + u_i \int d^d z \frac{C_{ij}}{|x-y|^{2\Delta_i-\Delta_j}|x-z|^{\Delta_j}|y-z|^{\Delta_j}} + \dots \quad (5.16)$$

There are two types of operators O_j that produce slow transients as the UV fixed point is approached. The first are almost marginal operators, $\Delta_j \sim d$. The intuition here is clear. Under the renormalization group flow it takes a long time for the perturbation to the action to fade away. This can be seen by evaluating the integral in 5.16. It is UV and IR convergent and so its value can be determined by scaling. Absorbing a constant we find

$$\langle O_i(x)O_i(y) \rangle = \frac{1}{|x-y|^{2\Delta_i}} (1 + u_i |x-y|^{d-\Delta_j}) + \dots \quad (5.17)$$

The UV fixed point can be seen in correlators where $|x-y| \ll L$. The approach to the fixed point is determined by the correction term in 5.17 which is $\sim |\frac{x-y}{L}|^{d-\Delta_j}$ where the characteristic size of u_j has been inserted.

A second source of slow transients are highly relevant low dimension operators of the kind discussed above. Here $\Delta_j \ll d$. At first glance it is surprising that these would give rise to slow transients. Their strength in the action rapidly decreases in the flow to the UV. But their effect on correlation functions is enhanced by IR divergences. In particular for $\Delta_j < d/2$ the integral in 5.16 is IR divergent. Keeping the leading IR divergent term in 5.16 we obtain

$$\langle O_i(x)O_i(y) \rangle = \frac{1}{|x-y|^{2\Delta_i}} (1 + u_i |x-y|^{\Delta_j} L^{d-2\Delta_j}) + \dots \quad (5.18)$$

Inserting the characteristic size of u_j we find a slow transient of order $\sim |\frac{x-y}{L}|^{\Delta_j}$. The slow transients in the tree model produced by the low dimension multiverse fields are of this type. Higher order terms in u_j do not alter these qualitative behaviors.

Leading order perturbative correlations in the doubled dS/CFT contain conjugate pairs of operators with dimensions Δ and $d - \Delta$. For small mass bulk scalars one of these is highly relevant, the other almost marginal. The above analysis shows that they contribute

the same type of slow transient in the flow to the UV. At higher orders in bulk perturbation theory the precise pairing is broken [26, 27]. The multiverse fields are highly nongaussian [19] but for low nucleation rates it seems plausible that the full doubled theory will contain nearly marginal as well as highly relevant operators.

As mentioned earlier, the non-perturbative effects induce configurations on Σ which can reasonably be called instantons. It is the role of instantons to mix otherwise inaccessible regions of target space. Moreover small instantons can induce non-perturbative RG flows. From the FRW/CFT point of view that is exactly what is happening.

6 Terminals and BPS Domain Walls

There are a number of reasons to believe that de Sitter space cannot be absolutely stable and must decay to terminal vacua. In particular there is a rigorous no-go theorem stating that the symmetries of de Sitter space cannot be realized in a system with finite entropy [28]. That theorem applies to dS_{2+1} .

In the usual framework of $(3 + 1)$ -dimensional de Sitter space, the way out of the theorem is the existence of terminals, which as shown in [19] and reviewed in section 9, breaks the de Sitter symmetries. The same mechanism should apply in the lower dimensional case, and indeed we believe it does.

The candidates for $(2 + 1)$ -dimensional terminals are BPS domain walls which can form when a supersymmetric ADS vacuum collides with the census taker's bubble [12]. The geometry of the BPS domain wall is flat. In fact, in the lower dimensional sense the BPS domain wall is a terminal $(2 + 1)$ -dimensional FRW geometry. To put it another way, the BPS domain walls provide exact $(2 + 1)$ -dimensional analogs of the $(3 + 1)$ -dimensional supersymmetric hat-geometries.

The BPS domain walls do not destroy the $(3 + 1)$ -dimensional census taker. In this respect the conformal diagram Figure 7 is misleading. Instead we can consider a metrically more faithful version in which at late-time, region I becomes the interior of a Minkowski-space future light cone. The BPS domain walls are non-accelerating planes that typically never get very deep into the region near $R = 0$. This is shown in Figure 12.

Their effect is precisely the lower dimensional analog of the effects discussed in [19] and Section 9. The rate equation 5.13 gets an additional term as in 5.9,

$$\frac{dP_n}{d\omega} = \sum_m (\gamma_{nm} P_m - \gamma_{mn} P_n) - \gamma_n P_n \quad (6.1)$$

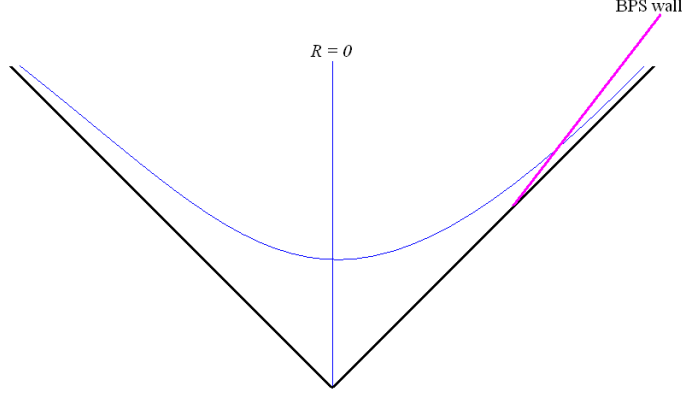


Figure 12: A BPS domain wall is a non-accelerating plane in the flat FRW geometry.

Again the leading time dependence is governed by a dominant eigenvector with eigenvalue $e^{-\Delta_D}$ ($\Delta_D > 0$) and the fractional coordinate area occupied by non-terminals goes to zero as $e^{-\Delta_D \omega}$. However the proper area of the non-terminal region increases as $e^{(2-\Delta_D)\omega}$.

Once there are terminals it becomes important to specify an initial condition on some spacelike hypersurface in the multiverse. We will follow Garriga, Guth, and Vilenkin [21] and define the initial condition by specifying an initial surface with a given color and no terminal bubbles present. The particular hypersurface breaks the symmetry of de Sitter space by singling out a preferred frame, namely the frame of the initial surface. If terminal vacua did not exist, that symmetry breaking would heal itself with time, but in the presence of terminals the symmetry breaking persists.

Conditional questions, of the kind described by 5.11, concerning the inflating fractal can be asked. The eigenvalues of 5.9 are all less than 1. The stationary eigenvector is replaced by a *dominant* eigenvector with a very small eigenvalue $e^{-\Delta_D}$. In addition there remains a discretuum of very small sub-leading scaling dimensions Δ_I , all satisfying $1 \gg \Delta_I > \Delta_D$.

As in Section 9, terminals destroy the conformal field theory that lives on Σ , but leave well defined conditional correlation functions. Given two points σ_1 and σ_2 , the conditional probability that they have color n_1 and n_2 , given that neither has been swallowed up by a terminal bubble, is defined to be

$$C_{n_1 n_2}(Y_1, Y_2). \quad (6.2)$$

The conditional correlations are scale invariant but not conformally invariant. In particular the two point function 6.2 scales but is non-diagonal in the dimensions of the operators.

This shows that the special conformal transformations that hold two points fixed on Σ are no longer symmetries. The symmetries that do remain (the symmetries of the conditional correlations) when BPS terminals are included are,

- Scale Transformations $Y^i \rightarrow \lambda Y^i$.
- Rotations of the Y plane.
- Translations of the Y plane.

It is clear that the effect of terminals is to change the attractor behavior to something new. In [19] we called it a *fractal flow*. The fractal flow is a universal attractor with correlation functions that scale, but it is not a field theory in any ordinary sense. Among other things, it lacks standard cluster-decomposition properties. It does have one remarkable property that we will come back to: unlike an conformal fixed point, the fractal flow leads to an arrow-of-time. In the present context it means that the lower-dimensional $2 + 1$ -dimensional geometry also has an arrow-of-time.

The breaking of conformal invariance by BPS domain walls is perhaps most obvious from the dS_{2+1} description of region II but it also has implications for the FRW/CFT duality in region I. The breakdown of symmetry is a form of persistence of memory described in [21]. There are two time-scales that relate to the persistence of memory. Consider the global initial condition for the multiverse, defined as a surface on which there are no terminal bubbles. By convention let the time of that surface be $t = 0$. The census taker's bubble nucleates at a later time, t_n . Inside the Census taker's bubble, the FRW time from the nucleation event can be called t . The persistence of memory states that for fixed t , there is a breaking of spatial homogeneity ($O(3, 1)$ symmetry) in the FRW patch that does not disappear as $t_n \rightarrow \infty$. This effect is a consequence of terminal vacua in the ordinary $(3 + 1)$ -dimensional sense. It does not in itself imply that the violation of spatial homogeneity persists to arbitrarily large FRW time $t \rightarrow \infty$ within the census taker's bubble. If the theory on Σ runs to a conformally invariant fixed point then the $O(3, 1)$ symmetry will be restored.

On the other hand the added ingredient of BPS domain walls breaks the $O(3, 1)$ of Σ . This is a stronger persistence that implies that the symmetry between census takers in the FRW patch is never restored as $t \rightarrow \infty$. There is a preferred census taker who can claim to be at the center of the spatial geometry.

The census taker at $R = 0$ may be special, but he is in the minority. All but a negligible fraction of census takers are out at asymptotic R . Therefore the limiting coordinates U^\pm, Y^i

are especially appropriate for polling the census takers in a given FRW cosmology. There left-over symmetry is sufficient to make sure that asymptotic Census Takers at large R are all equivalent.

Finally it is obvious that the radiation entering the hat will be affected by terminal bubble nucleation, replacing the fixed point with something a little different.

7 Region III

In this paper we have had almost nothing to say about region III other than that it exists and borders on Σ . The most interesting questions are about region III and its relation to the census taker. What is clear is that just as the surface **a** defines initial conditions for the FRW region, the surface **b** defines initial conditions for the rest of the multiverse: region III. Obviously the phenomena that take place in region II create initial conditions for both I and III.

From the point of view of the census taker region III is analogous to the region behind the horizon with the census taker playing the role of an observer who remains outside the black hole. As in the black hole case, the census taker receives Hawking radiation from the horizon, i.e., the surface that we earlier called **b**. Unlike the black hole case, the amount of radiation is infinite, which raises the question of whether it can encode the entire multiverse. Obviously we think the answer may be yes. One way of expressing the issue is to ask if the Hawking radiation is complementary to information in region III, and if so, can complementarity provide a tool for addressing the measure problem?

Complementarity requires that the information in region III can be “pulled back” through the horizon to region II, and then run forward into region II. It is not obvious that all information in III can be pulled back in this way. Just to illustrate, consider figure 13. For a story A located in a generic point in region III, the portion of its causal past represented by light blue, does not intersect the census taker’s horizon. However, it is also clear from the figure, that as A is moved closer to Σ , the missing fraction of information tends to zero. That is one of the reasons for zooming in on Σ as we did in Section 1. In that limit no information gets to A without passing through the horizon. Thus, in the limit, it should be possible to pull every story back into region II. Since the entire multiverse is replicated in every small comoving volume, zooming in should not be a real limitation in studying the rest-of-the-multiverse.

The problem of how horizons encode, or perhaps encrypt is a better word, information behind horizons is a new kind of quantum-information-theoretic question which is receiving

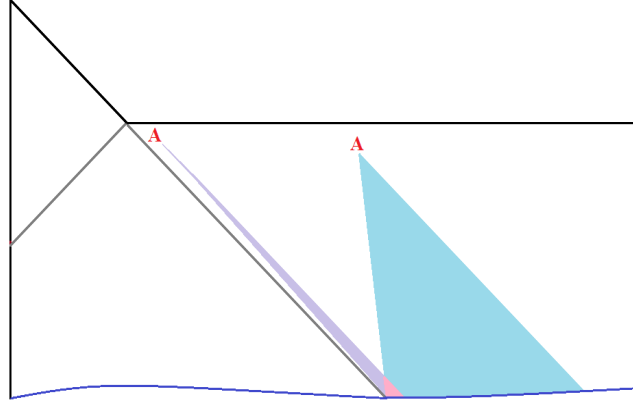


Figure 13: As the story A is moved toward Σ all but a vanishingly small fraction of its information passed through the horizon. The dark blue line at the bottom of the figure represents the global initial condition.

recent attention [29] [30] [31].

8 Conclusions

Let us list and review the conclusions of this investigation:

- The first point—one which we have emphasized in earlier publications—is that a precise formulation of quantum-gravitational systems requires the existence of a causally connected region which has access to an infinite amount of information [7]. This precludes conventional closed FRW cosmologies, and stable eternal de Sitter vacua. If one adds the condition [4] that irreversible de-coherence can occur, that leaves eternal inflation as the only candidate. The causally connected regions with infinite information are the supersymmetric FRW geometries bounded by hats. The holographic description of such a region is the FRW/CFT theory located on the boundary-surface Σ .
- The surface Σ is more than just the location of the FRW/CFT holographic duality. It is the intersection of three regions. Region I is the census taker's FRW region

bounded by a hat. Region II is a $(2 + 1)$ -dimensional de Sitter space called dS_{2+1} . Region III is the rest of the multiverse behind the census taker's horizon.

- Unlike the usual global $(3 + 1)$ -dimensional de Sitter space, the entire future boundary of dS_{2+1} can be seen by a single observer. In that sense the theory on Σ has operational significance that a theory on the future boundary of $(3 + 1)$ -dimensional de Sitter space would not have. The census taker plays the role of a meta-observer for dS_{2+1} .
- While dS_{2+1} has a finite de Sitter entropy, the total amount of information visible to the census taker is infinite. The meaning of this unbounded amount of information is not certain, but we conjecture that it is complementary to the unbounded information in region III—the rest of the multiverse.
- In perturbation theory, the FRW/CFT fixed point on Σ is specific to a particular Coleman De Luccia decay. However, non-perturbative effects of bubble collisions lead to a mixing of the initial vacua and the creation of a more universal fixed point that does not depend on the initial ancestor vacuum.
- dS_{2+1} is a genuine quantum de Sitter space with a dynamical gravitational field. It defines a lower dimensional version of eternal inflation. In the absence of terminal vacua the 2-dimensional dS/CFT theory living on Σ is conformally invariant.
- BPS domain walls connecting the census taker's vacuum with supersymmetric ADS bubbles correspond to terminal vacua in the $(2 + 1)$ -dimensional eternally inflating theory in dS_{2+1} . Such terminals break the conformal invariance on Σ , which in turn means that spatial homogeneity is broken in the census taker's FRW patch. This is a strong form of persistence of memory and lead to a preferred spatial center in FRW, for all time.
- The existence of large number of $(3 + 1)$ -dimensional de Sitter vacua leads to an equally large number of very low dimensional multiverse fields. These multiverse fields are very relevant in the RG sense. Therefore the flow to the infrared can be expected to break up into a huge number of branches. These branches correspond to different initial starting vacua for eternal inflation. This may provide a way of classifying vacua by operators in the FRW/CFT theory.

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9 Appendix

In this appendix we derive the relationship between the nucleation time of a bubble of some dS vacuum B in region II and its size on the census-taker’s late-time sky. The metric of the ancestor dS space in flat slicing is

$$ds^2 = \frac{1}{T^2} (-dT^2 + d\vec{x}^2). \quad (9.1)$$

We can take the census taker’s bubble to nucleate at $\vec{x} = 0$, $T = -1$, and we will take the bubble of vacuum B to nucleate at $\vec{x} = \vec{x}_B$, $T = T_B$. These nucleations will remove the union of two spheres from future infinity in the ancestor dS space, one centered at $\vec{x} = 0$ with radius one and the other centered at $\vec{x} = \vec{x}_B$ with radius $-T_B$. The affected angular size on the census-taker’s late-time sky is shown in figure 14 and is given by

$$\cos\left(\frac{\theta}{2}\right) = \frac{1 - \vec{x}_B^2 + T_B^2}{2|\vec{x}_B|}. \quad (9.2)$$

We’d like to recast this in terms dS slicing coordinates in region II, in terms of which the metric is

$$ds^2 = d\xi^2 + \sin^2 \xi [-d\omega^2 + \cosh^2 \omega d\Omega_{d-1}^2]. \quad (9.3)$$

By looking at the relationship of the coordinates (9.1) and (9.3) to the embedding coordinates in $d + 2$ dimensional Minkowski space we find

$$\begin{aligned} \sin \xi \sinh \omega &= \frac{1}{2} \left(T - \frac{1 + \vec{x}^2}{T} \right) \\ \sin \xi \cosh \omega &= -\frac{|\vec{x}|}{T}, \end{aligned} \quad (9.4)$$

whose ratio then gives

$$\tanh \omega = \frac{1 + \vec{x}^2 - T^2}{2|\vec{x}|}.$$

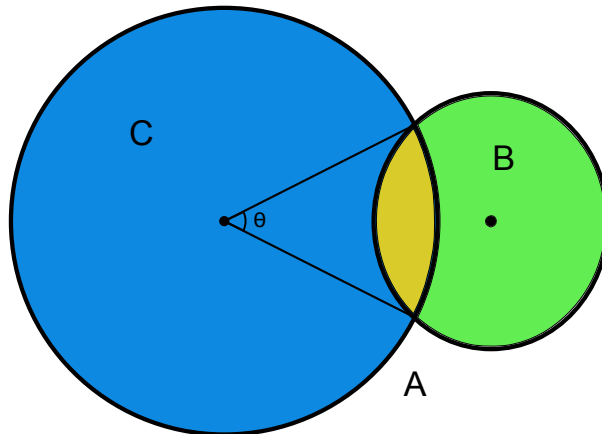


Figure 14: The region subtended at future infinity by a collision of a census-taker bubble C with a dS bubble B inside of an ancestor vacuum A . What happens inside the yellow region depends on the details of the domain wall between B and C , but it does not affect the angle the collision influences on the census-taker’s sky, which is denoted θ .

From (9.2) we thus see that the angle subtended on the census-taker’s sky is simply related to the time in the dS slicing via

$$\cos\left(\frac{\theta}{2}\right) = \tanh \omega_B.$$

Interestingly the angle is independent of the spatial coordinate ξ , so the dS slicing is particularly natural for talking about how nucleation events in region II affect the census-taker’s observations.

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